York University
EECS 2011Z Winter 2015 – Problem Set 1
Instructor: James Elder

This problem set will not be graded, but will help you understand asymptotic analysis of running times, and thus prepare for the midterm. You are free to work together on these if you prefer. Solutions will be posted Thursday, Jan 29. There are also problems at the end of Ch. 4 of the textbook if you would like additional practice.

1. Prove whether each of the following is true or false. x and y are real variables.

   1) $\forall x \ \exists y \ x \cdot y = 5$
   2) $\exists y \ \forall x \ x \cdot y = 5$
   3) $\forall x \ \exists y \ x \cdot y = 0$
   4) $\exists y \ \forall x \ x \cdot y = 0$
   5) $\exists a \ \forall x \ \exists y \ [x = a \text{ or } x \cdot y = 5]$

2. Asymptotic Running Times

   True or False? All logarithms are base 2. No justification is necessary.

   (a) $5n^2 \log n \in O(n^2)$
   (b) $4^{8n} \in O(8^{4n})$
   (c) $2^{10 \log n} + 100(\log n)^{11} \in O(n^{10})$
   (d) $2n^2 \log n + 3n^2 \in \Theta(n^3)$

3. Big-Oh Definition

   Fill in the blanks:

   $f(n) \in O(g(n))$ iff ____ $c > 0$, ____$n_0 > 0$, such that ____ n ____n_0, $f(n)$____ $cg(n)$

4. Order the following functions by increasing asymptotic growth rate:

$$\begin{array}{ccc} 4n\log n + 2n & 2^{10} & 2^{\log n} \\ 3n + 100\log n & 4n & 2^n \\ n^2 + 10n & n^3 & n\log n \end{array}$$

5. Prove that $n \log n - n$ is $\Omega(n)$.

6. Prove that if $d(n)$ is $\mathsf{O}(f(n))$ and $e(n)$ is $\mathsf{O}(g(n))$, then the product $d(n)e(n)$ is $\mathsf{O}(f(n)g(n))$.

7. An evil king has n bottles of wine, and a spy has just poisoned one of them. Unfortunately, they dont know which one it is. The poison is very deadly; just one drop diluted even a billion to one will still kill. Even so, it takes a full month for the poison to take effect. Design a scheme for determining exactly which one of the wine bottles was poisoned in just one months time while expending only $\mathsf{O}(\log n)$ royal tasters. State your scheme briefly, in English.

8. Asymptotic Running Times

   True or False? All logarithms are base 2. No justification is necessary.

   (a) $2^n \in \Omega\left(n^3\right)$
   (b) $3n^3 + 17n^2 \in O\left(n^3\right)$
   (c) $5n^2 \log n \in O(n^2)$
   (d) $2^{10 \log n} + 100(\log n)^{11} \in O(n^{10})$
   (e) $2n^2 \log n + 3n^2 \in \Theta(n^3)$

9. Show that n^2 is $\Omega(n \log n)$.